The Association
of
Engineering and Shipbuilding
Draughtsmen.

GEAR MANUFACTURE.

BOOK 3.—SPIRAL AND HELICAL GEARS.

Detail Design.
Form Milling.
Hobbing.
Rack Planing.
Gear Shaping.
Profile Grinding.

By F. WHITELEY.

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Also published—

"GEAR MANUFACTURE"

By F. Whiteley.

BOOK I.

Spur gears—internal spur gears—spur rack—helical gears—double helical gears—spiral gears—bevel gears—spiral bevel gears—worm gears—notations—involite form—construction of involute form—generation of tooth form—involute function—correction of spur gears—stub tooth—increasing pressure angle—positive and negative correction—extended centre distance gearing—involute tooth action—specific sliding. (30 diagrams).

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BOOK 2.

Spur gears—detail design—addendum and dedendum—pitch—pressure angle — backlash — vernier settings — constant chord settings—form milling—dividing head—differential indexing—hobbing—shaping—rack planing—profile grinding—formers—generation. (22 diagrams).

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BOOK 3.

SPIRAL AND HELICAL GEARS.

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GEAR MANUFACTURE.

By F. WHITELEY.

CHAPTER I.

DETAIL DESIGN.

THE design of helical gears differs from the design of spur gears, only in so far as the teeth instead of being parallel to the axis are of helical or screw form, the spiral angle denoting the angular difference between the axis and the inclination of the teeth.

For helical gears the spiral angle need only be sufficient to provide for continuous action as any angle greater than this only increases the end thrust without obtaining any gain in efficiency or quietness of action.

Therefore the minimum spiral angle of a pair of helical gears can be calculated as follows :—

Let F = face width of gear.
ϕ = spiral angle of gear.
p_t = circular pitch on transverse plane.

$$\text{Then } \tan \phi = \frac{p_t}{F}$$

or in words the minimum spiral angle equals the advance of one tooth in width of face to ensure continuous action.

The chief advantage secured by using a helical gear in place of a spur gear is that the line of contact is progressive, commencing at the tip of the tooth and terminating at the base circle on one tooth, and the reverse on the mating tooth, that is, it commences at the base circle and terminates at the tip.

The minimum spiral angle can, of course, be easily calculated when the transverse pitch is standard as is the case when using the gear shaper method for cutting the gear teeth, and, in fact, a pair of spur gears can be replaced by a pair of helical gears without any trouble when the circular pitch or the pitch on the transverse plane of the helical gears is the same as the spur gears which are being replaced.

However, the majority of helical and spiral gears are produced by the hobbing or rack planing methods in which case the normal pitch is generally made standard, and not the transverse pitch.

In this case, therefore, as the spiral angle increases the addendum and dedendum decrease in terms of the transverse pitch, while for the gear shaper method for helical gears the stub tooth is adopted to compensate for the introduction of the spiral angle. Thus for 5 DP circular pitch the addendum and dedendum are made to 7 DP proportions and denoted by 5/7 DP, but this feature of the helical gear is explained under the heading "Gear Shaping."

The most important factor therefore when dealing with any calculations for helical or spiral gears is to be able to distinguish between the normal, transverse, and axial pitches, and the definition of these is given below.

Normal Pitch.

The normal pitch is the circular distance measured on the pitch circle, between similar faces of successive teeth taken on a helix lying on the pitch circle at right angles to the direction of the teeth and denoted as p_n.

For all gears, excepting helical gears produced by the gear shaper process, the normal pitch is usually specified in terms of DP therefore in inches this will equal $\frac{\pi}{DP}$.

Should the transverse pitch be specified in the case of helical gears cut on the gear shaper then

$$p_n = p_t \cos \phi \text{ when } \phi = \text{spiral angle.}$$

Transverse Pitch.

The transverse pitch is the circular distance measured on the pitch circle between similar faces of successive teeth taken on the plane of rotation or at 90° to the axis of the gear, and denoted by p_t.

Given the normal pitch p_n then transverse pitch $p_t = p_n \sec. \phi$ when ϕ = spiral angle.

Axial Pitch.

The axial pitch is the lineal distance measured on the pitch circle between similar faces of successive teeth on a plane parallel to the axis of the gear and denoted by p_a.

The axial pitch can be calculated in two ways, first given the normal pitch in inches then $p_a = \frac{p_n}{\sin \phi}$ when ϕ = spiral angle

secondly $p_a = \frac{L}{Z_1}$

When p_a = axial pitch.
p_n = normal pitch in inches.
ϕ = spiral angle on diameter D.
L = lead of spiral.
Z_1 = number of teeth in gear.
D = pitch circle diameter.

when the lead of spiral L is first obtained from the formula.

$$L = \frac{D\pi}{\tan\phi}$$ from which it will be noted that

$L = Z_1 \times p_a$ when Z_1 is equal to the number of teeth in the gear.

To illustrate the relationship of the various pitches to each other a cylinder has been drawn in Fig. 1 representing the pitch cylinder of a helical or spiral gear and around the cylinder two helixes have been drawn representing two rings of teeth, the normal, transverse, and axial pitches being clearly shown.

The lead of spiral is also indicated in relation to the pitch cylinder D and the axial pitch p_a while the spiral angle defines the position of the angle of tooth spiral on the pitch cylinder D.

Tooth Proportions.

The standard tooth proportions of spiral and helical gears cut by any method other than the gear shaper method are based on the full depth involute system in terms of the normal pitch.

Then addendum $= \dfrac{1}{DP_n}$

dedendum $= \dfrac{1 \cdot 157}{DP_n}$

tooth thickness $= \dfrac{\pi}{2\,DP_n}$

When DP_n is the normal pitch expressed in terms of diametral pitch.

Vernier Settings.

The tooth thickness can only be measured using a pair of gear tooth verniers on the *normal* plane, *i.e.*, at right angles to the tooth spiral, and it is correct to use pitch line settings only.

Constant chord settings should not be used on the normal section as the tooth form on this plane is not of strictly correct involute form, the only plane on a helical or spiral gear which is of pure involute form being the transverse plane for generated gears.

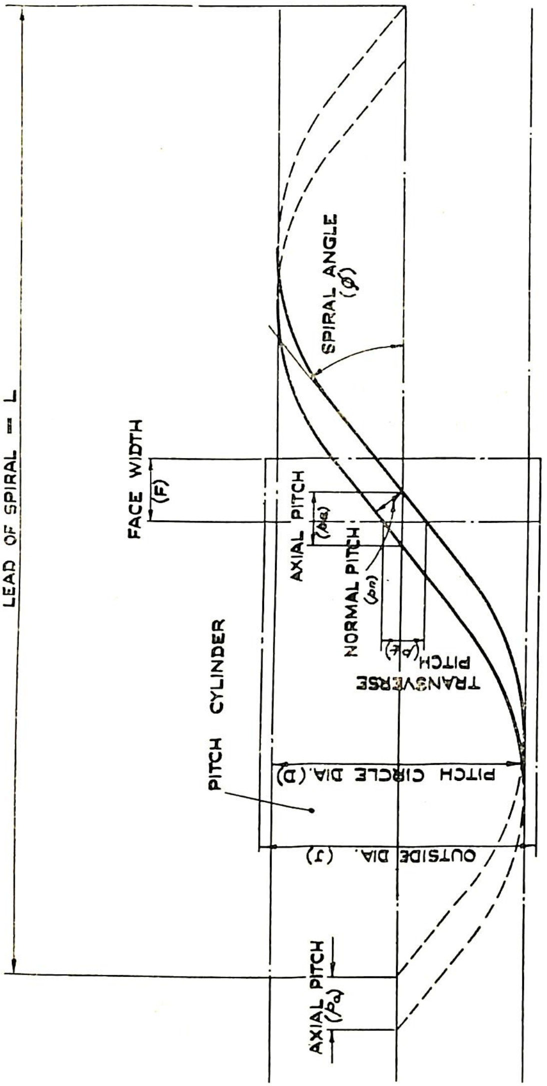


FIG. 1. SHEWING RELATIONSHIP OF NORMAL TRANSVERSE AND AXIAL PITCHES TO LEAD OF SPIRAL AND PITCH CIRCLE DIAMETER

This fact is very important when dealing with gears of large spiral angles, and if the shape of tooth on the normal plane is required then co-ordinates have to be obtained from the transverse plane, and transferred to the normal plane, but this will be dealt with later.

Referring to Fig. 2 it will be noted that the pitch circle viewed on the normal plane is distorted, and is, in fact, part of an ellipse, the major radius of this ellipse being termed the virtual pitch radius.

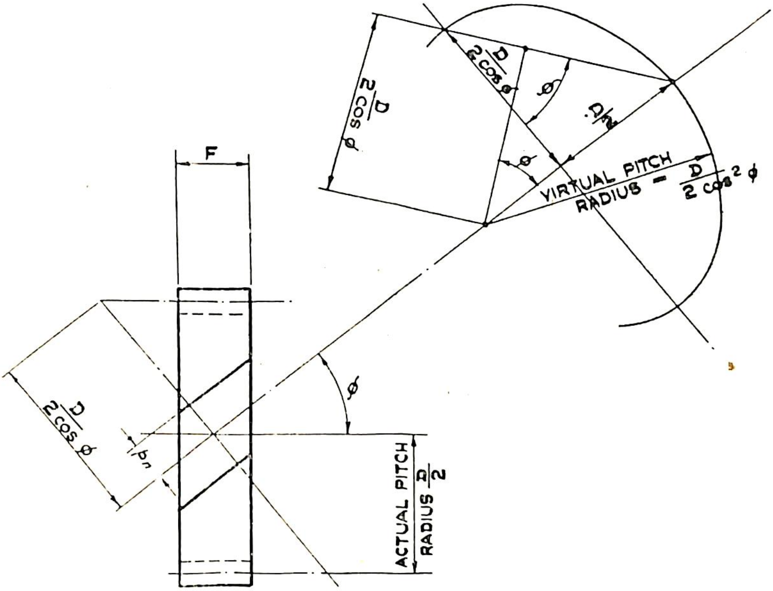


FIG. 2. CONSTRUCTION OF VIRTUAL PITCH RADIUS ON NORMAL PLANE.

For purposes of calculation, however, this can be expressed as a virtual pitch diameter and is, in fact, the pitch diameter of a spur gear having an equivalent number of teeth the same pitch as the normal pitch of the spiral or helical gear and expressed as a formula we have :—

$D_v = \frac{D}{\cos^2\phi}$ and the virtual or equivalent number of teeth which will correspond to D_v of the same normal pitch as the spiral gear

$$Z_v = \frac{Z}{\cos^3 \phi}$$

When D = actual pitch diameter of gear on transverse plane.
D_v = virtual pitch diameter on normal plane.
ϕ = spiral angle.
Z = actual number of teeth in gear.
Z_v = virtual or equivalent number of teeth on normal plane.

The values D_v and/or Z_v are then used to obtain the pitch line vernier settings in exactly the same manner as was used for spur gears as follows :—

If the value Z_v is used then it is only necessary to refer to the vernier pitch line settings for spur gears of 1 DP pitch given in Book II, Chapter 1, letting Z_v equal the number of teeth and dividing the chordal addendum and chordal thickness by the normal diametral pitch, providing that the tooth section on the normal plane is of standard proportions.

The value D_v is used for gears of special proportions on the normal section, and can also be used for gears of standard proportions and referring to Fig. 3 let :—

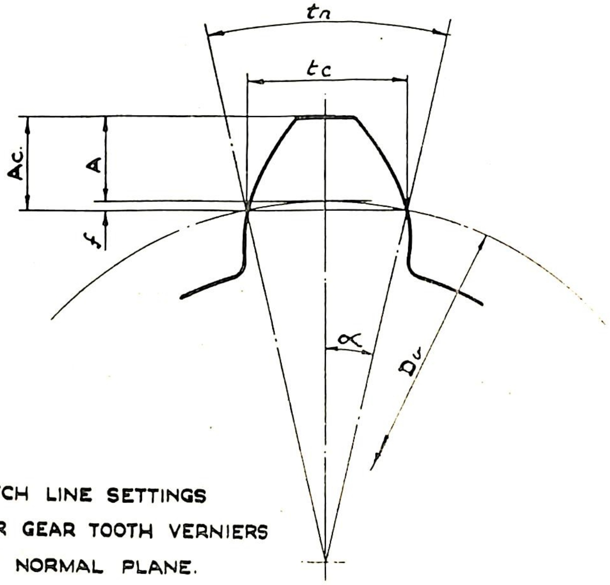


FIG. 3. PITCH LINE SETTINGS FOR GEAR TOOTH VERNIERS ON NORMAL PLANE.

D_v = virtual pitch circle diameter.
t_n = circular thickness of tooth on normal plane.
α = half-angle subtended by t_n.
A = addendum of gear.
f = correction factor.
A_c = chordal addendum on normal plane.
t_c = chordal thickness on normal plane.

Then given t_n

$$\alpha \text{ radians} = \frac{t_n}{D_v}$$

Convert α radians to degrees when one radian = 57·296 degrees.
then $\sin \alpha \, D_v = t_c$

$$\text{and } f = \frac{D_v}{2} \text{ versine } \alpha$$

$$\text{Then } A_c = A + f$$

$$\text{When } A = \frac{J - D}{2}$$

Backlash.

From the value t_c must be deducted half the backlash allowance, and this may be the same as that given for spur gears in the table given in Book II, Chapter 1, the DP pitch given being substituted by the normal diametral pitch DP_n. Thus the backlash on the normal section will be the same as that given in the table for spur gears of the same pitch, although when using the table for gears having large spiral angles, it is advisable to reduce the allowance to avoid excessive rotary movement of the gears on the transverse plane.

Pressure Angle.

In the majority of cases it will be found that generated gears of standard proportions can be used without recourse to correction, excepting gears produced by the gear shaper method, which being standard on the transverse plane must be treated as spur gears of the same proportions.

For spiral and helical gears produced by other processes, however, the design should satisfy the following conditions before any tooth correction is necessary.

When Z_v = 32 teeth and over, make $14\frac{1}{2}°$ normal pressure angle.
When Z_v = 17 teeth and over, make 20° normal pressure angle.
When Z_v = 11 teeth and over, make 25° normal pressure angle.

The normal pressure angle given above being, of course, the pressure angle of the rack planing cutter or hob and Z_v being the virtual number of teeth, and not the actual number of teeth in the gear.

To obtain the pressure angle of the gear on the transverse plane or the plane on which the true involute is being produced, we proceed as follows :—

Let ψ_n = pressure angle of cutter.
ψ_t = transverse pressure angle of gear.
ϕ = spiral angle.

$$\text{Then } \tan \psi_t = \frac{\tan \psi_n}{\cos \phi}$$

This pressure angle is most important when it is desired to obtain particulars of the tooth shapes on the normal and axial planes, due to the fact that it is only on the transverse plane where the tooth shape is of involute form.

The tooth space shape on the normal plane is usually required when the gears are being cut with the form milling cutter method, as the cutter form must be made to suit the normal tooth space. The method used to obtain this shape is explained in Chapter 2 on the subject of Form Milling Spiral and Helical gears.

The tooth space form on the axial section is required for the purpose of obtaining roller sizes for measuring the teeth over two rollers, and the method used is described later on the subject of Testing Spiral and Helical gears.

Base Circle Diameter.

The base circle diameter D_o or d_o of a spiral or helical gear is obtained from the transverse pressure angle when

ψ_t = transverse pressure angle of gear.
D or d = pitch circle diameter of wheel or pinion.
D_o or d_o = base circle diameter of wheel or pinion.
Then $D_o = D \cos \psi_t$ or $d_o = d \cos \psi_t$.

The base circle diameter is used when it is desired to check the involute tooth shape on the transverse plane, using an involute tooth testing machine, or when making a layout of the tooth shape on the transverse plane.

Base Lead Angle.

It is sometimes found necessary to calculate the base lead angle of a spiral or helical gear especially when the spiral angle is large, although the angle is used more often when checking the tooth shape of a worm.

Given ϕ_o = spiral angle of gear on base circle diameter.
D_o = base circle diameter.
L = lead of spiral.

$$\text{Then } \tan \phi_o = \frac{D_o \pi}{L}$$

and base lead angle
$= 90° - \phi_o$.

or cos base lead angle
$= \cos \psi_n \cos (90° - \phi)$.

When ψ_n = normal pressure angle.
ϕ = spiral angle on pitch circle diameter.

Spiral Gears.

The rules previously given are applicable to both helical and spiral gears except in the case of the width of face F and the spiral angle.

Helical gears meshing together in the same manner as spur gears are made the same spiral angle but opposite hand, as shown in Fig. 4, Book I, but as will be noted by referring to the spiral gears in Fig. 6, Book I, the gears are made the same hand, and the wheel and pinion may not always be the same angle of spiral.

If this is the case the value ϕ previously quoted must be substituted by the angle θ when

ϕ = spiral angle of pinion.
θ = spiral angle of wheel.

That is for helical gears ϕ will equal θ but for spiral gears gearing at 90° shaft angle ϕ will equal $90° - \theta$.

With regard to the rule given to obtain the minimum width of face F this only applies to helical gears, as a pair of spiral gears give only theoretical point contact and therefore the width of face is purely arbitrary.

In addition to this, very large spiral angles are common with the driving member of a pair of spiral gears, in fact the value can be made as high as nearly 90° if necessary, therefore the rule given for the width of face for helical gears would bring about an impossible condition, if used for spiral gears.

A most peculiar situation which can arise in connection with the design of spiral gears gearing at 90° to each other, is the one in which the pinion having a smaller actual number of teeth can still be greater in diameter than the wheel.

This is accomplished by making the spiral angle of the pinion greater than 45° and using the pinion as the driver while the wheel spiral angle is made (90°—pinion spiral angle).

This fact is very important when designing spiral gears, and in the majority of cases it is necessary to use "trial and error" methods for obtaining the correct pitch circle diameters and spiral angles, when the normal pitch and gear centres are fixed.

To avoid unnecessary calculations, however, the pitch circle diameters and spiral angles can be obtained practically correct from a layout generally as shown in Fig. 4, after which "trial and error" methods have to be resorted to for obtaining the absolutely correct values.

Referring to figure, lines OA and OB are first drawn at right angles to each other after which a distance OC is marked on line OA, this distance being made equal to $\frac{Z_1}{DP_n}$

Similarly a distance OD is marked on line OB, this distance being made equal to $\frac{Z_2}{DP_n}$ and the rectangle OC, OD, CE and DE completed.

Using the corner of the rectangle marked E as the axis, line GF which must equal twice the centre distance in inches, should be so swivelled that the line just reaches lines OA and OB and at the same time touches the corners of the rectangle E.

When this has been finally accomplished the pitch diameter of the pinion *d* can be measured from the layout, this distance being equal to EG while the distance EF will equal the pitch diameter of the wheel D, and similarly the spiral angles of the wheel and pinion θ and ϕ can also be obtained from the layout the angle θ being opposite line DF and angle ϕ opposite line CG.

When the approximate pitch diameters and angles have been obtained from the layout as described the particulars are checked from the formula.

$$D = \frac{Z_1}{DP_n \cos\theta} \text{ or } d = \frac{Z_2}{DP_n \cos\phi}$$

and $Z_1 = D \cos\theta\, DP_n$ or $Z_2 = d \cos\phi\, DP_n$.

When D = pitch circle diameter of wheel.

d = pitch circle diameter of pinion.

Z_1 = actual number of teeth in wheel.

Z_2 = actual number of teeth in pinion.

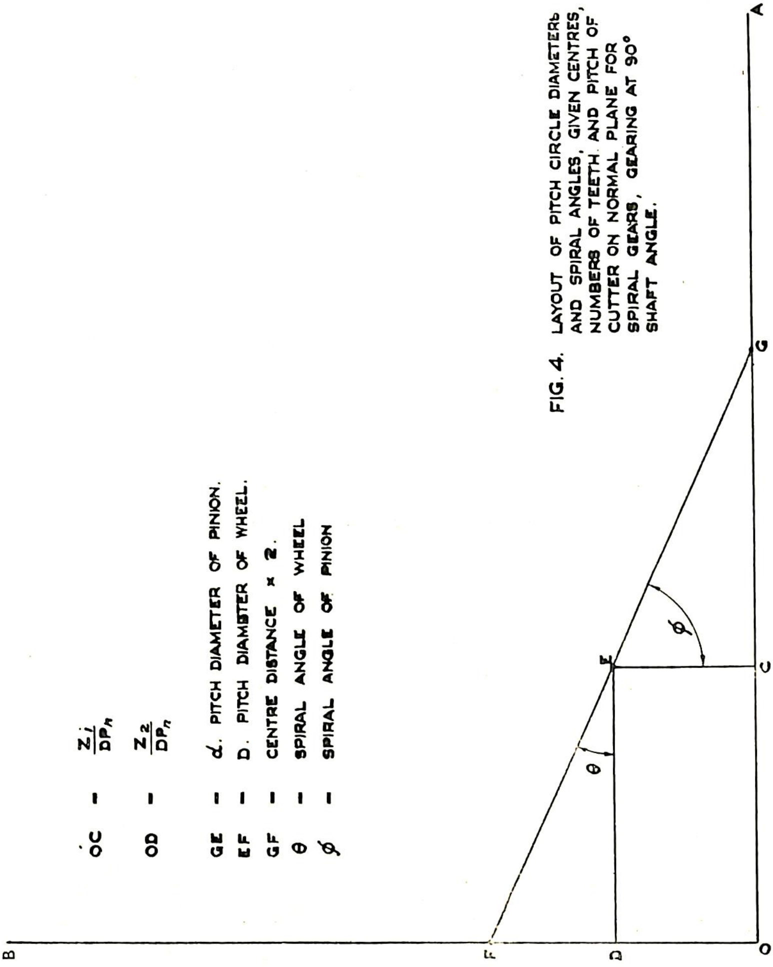
DP_n = normal pitch in terms of diametral pitch.

θ = spiral angle of wheel.

ϕ = spiral angle of pinion.

If the sum of the values D and *d* do not at the first attempt equal twice the centre distance, a slight adjustment to the angles θ and ϕ will be necessary, until the exact values D and *d* are obtained from the formula given.

More accurate results will, of course, be obtained from the layout, if this is made on an enlarged scale where practicable.



OC - $\frac{Z_1}{DP_n}$

OD - $\frac{Z_2}{DP_n}$

GE - d. PITCH DIAMETER OF PINION.

EF - D. PITCH DIAMETER OF WHEEL.

GF - CENTRE DISTANCE × 2.

θ - SPIRAL ANGLE OF WHEEL

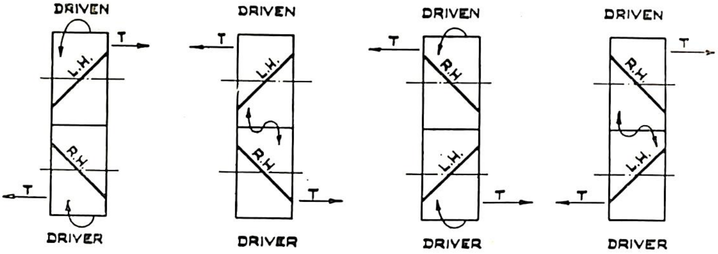
φ - SPIRAL ANGLE OF PINION

FIG. 4. LAYOUT OF PITCH CIRCLE DIAMETERS AND SPIRAL ANGLES, GIVEN CENTRES, NUMBERS OF TEETH, AND PITCH OF CUTTER ON NORMAL PLANE FOR SPIRAL GEARS, GEARING AT 90° SHAFT ANGLE.

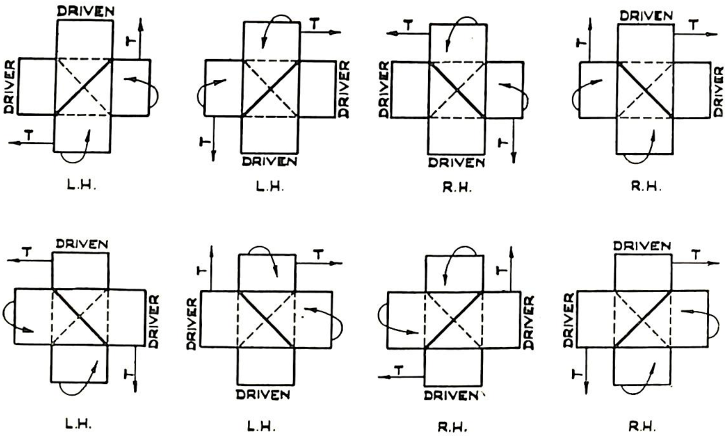
Shaft Angle and Thrust.

One of the most important points in connection with the design of spiral and helical gears is to be able to determine the direction of the end thrust which is set up by the introduction of the angle of spiral.

The hand of spiral, *i.e.*, left or right hand can be determined by reference to the diagrams shown in Fig. 5 in conjunction with the thrust, the arrows at the side of each gear indicating the direction of the thrust, and it will be noted that the direction of the thrust depends on the relative positions of the driver and driven gears, the direction of spiral, and the direction of rotation.



HELICAL GEARS. 'T' INDICATES DIRECTION OF THRUST



SPIRAL GEARS 90° SHAFT ANGLE 'T' INDICATES DIRECTION OF THRUST.
THRUST CAN BE REVERSED BY
(1) INTERCHANGING DRIVER OR DRIVEN GEAR.
(2) REVERSING DIRECTION OF ROTATION
(3) CHANGING DIRECTION OF TOOTH SPIRAL.

FIG. 5. THRUST DIAGRAMS

If the exact condition cannot be found in the diagrams it may be found by :—

(*a*) Interchanging the driver or driven gear.
(*b*) Reversing the direction of rotation.
(*c*) Changing the direction of spiral.

as any one of the above alterations will reverse the direction of thrust.

For spiral gears not gearing at 90°, it can be stated that as far as the calculations are concerned they are the same for a 135° shaft angle as for a 45° shaft angle, while the following general rule for gears having a shaft angle of 45° should be observed.

When the spiral angle of each gear is less than 45° the tooth spirals are of the same hand and one spiral angle (of either the wheel and pinion) is 45° minus the spiral angle of the other.

When, however, the spiral angle of either wheel or pinion is greater than 45°, the spiral angles are of opposite hand and the spiral angle of one gear is 45° plus the spiral angle of the other.

For spiral gears having their shafts at any given angle, the following rules should be observed.

If each spiral angle is less than the shaft angle the sum of the spiral angles of the two gears must equal the shaft angle and the hand of spiral must be the same for both gears.

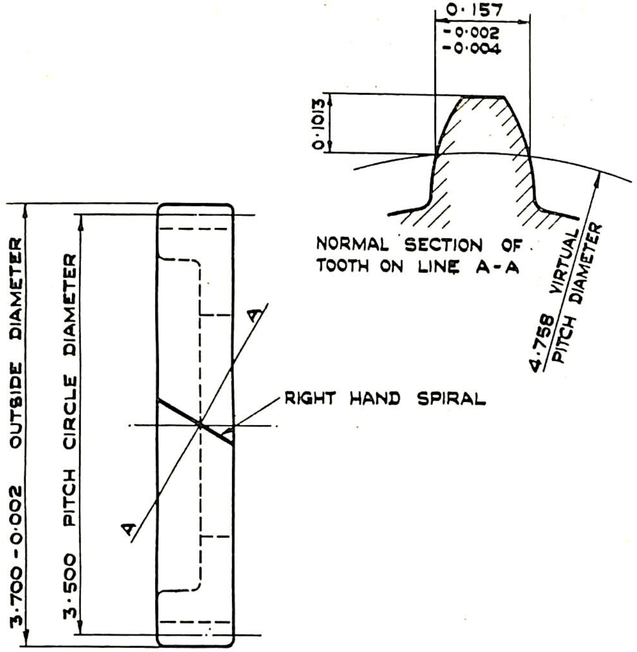
If, however, the spiral angle of one of the gears is greater than the shaft angle the difference between the spiral angles of the two gears must equal the shaft angle, and the gears must be made opposite hand to each other.

It is of considerable advantage to first draw the outline of one gear indicating the spiral angle and direction of spiral, and on a sheet of tracing paper the outline of the mating gear should be drawn.

If this is done the outline of the gear on the tracing paper can be moved about in relation to the first gear, the tooth spiral traced through and the conditions of thrust, rotation, and angle of spiral determined.

Example.—Given below are examples of first a helical gear and secondly a pair of spiral gears, gearing together at 90° shaft angle.

In either case a drawing of each gear should be made, the cutting particulars being neatly tabulated generally as shown in Fig. 6, and the same particulars will apply to both helical and spiral gears as they can both be cut in the same manner, except of course the helical gears which can also be produced by the gear shaper method.


0·157
-0·002
-0·004
0·1013
NORMAL SECTION OF TOOTH ON LINE A-A
4·758 VIRTUAL PITCH DIAMETER
3·700 - 0·002 OUTSIDE DIAMETER
3·500 PITCH CIRCLE DIAMETER
A
A
RIGHT HAND SPIRAL

HELICAL GEAR

CUTTING PARTICULARS	
TYPE OF GEAR	HELICAL
ACTUAL NUMBER OF TEETH	30
ADDENDUM	0·100
DEDENDUM	0·116
SPIRAL ANGLE	30° - 57′
NORMAL PITCH	10 DP
TRANSVERSE PITCH	0·3663
HAND	RIGHT
LEAD OF SPIRAL	18·336
NORMAL PRESSURE ANGLE	20°
TRANSVERSE PRESSURE ANGLE	22°-59′-44″
NORMAL CIRCULAR THICKNESS	0·1571
VIRTUAL NUMBER OF TEETH	48
RUNNING CENTRES	3·500 + 0·003
WITH MATING GEAR	TOGETHER
BACKLASH	0·004/0·008
CUTTER OR HOB. No	STD HOB
PRESSURE ANGLE	20°
PITCH OF CUTTER OR HOB	10 DP

DRG No S162

FIG. 6. WORKING DRAWING OF HELICAL GEAR.

Example No. 1—Helical Gears.

Helical gears having 30 teeth 1·000 width of face to gear with mating gear having 30 teeth at 3·500 centres, to be produced by hobbing or rack planing method.

$$\text{Module on transverse plane} = \frac{2\,C}{Z_1 + Z_2} = \frac{2 \times 3{\cdot}500}{30 + 30}$$

$$\begin{aligned} &= 0{\cdot}1166 \text{ and} \\ p_t &= 0{\cdot}1166 \times 3{\cdot}1416 \\ &= 0{\cdot}3663 \end{aligned}$$

Then minimum spiral angle ϕ to give progressive contact is equal to

$$\tan\,\phi = \frac{p_t}{F} = \frac{0{\cdot}3663}{1{\cdot}000} = 20^\circ\ 7'$$

and $p_n = p_t \cos\phi = 0{\cdot}3663 = 0{\cdot}93899 = 0{\cdot}34395$

The nearest standard diametral pitch smaller than 0·34395 is then taken to ensure full progressive contact, and in this case we take 0·3146.

Then normal pitch in terms of DP_n

$$= \frac{3{\cdot}1416}{0{\cdot}31416} = 10\ \text{DP}$$

and new spiral angle using 10 DP cutter equals

$$\cos\,\phi = \frac{p_n}{p_t} = \frac{0{\cdot}31416}{0{\cdot}3663} = 0{\cdot}85765 = 30^\circ\ 57'$$

$$\text{Then lead } L = \frac{D\,\pi}{\tan\,\phi}$$

$$\text{when} \quad D = \frac{p_t\,Z_1}{\pi} = \frac{0{\cdot}3663 \times 30}{3{\cdot}1416} = 3{\cdot}500$$

$$\text{Then} \quad L = \frac{3{\cdot}500 \times 3{\cdot}1416}{0{\cdot}59967} = 18{\cdot}336$$

and axial pitch

$$p_a = \frac{L}{Z_1} = \frac{18{\cdot}336}{30} = 0{\cdot}6112$$

$$\text{addendum A} = \frac{1}{DP_n} = \frac{1}{10} = 0{\cdot}100$$

$$\text{dedendum B} = \frac{1{\cdot}157}{DP_n} = \frac{1{\cdot}157}{10} = 0{\cdot}116$$

and outside diameter J

$$= D + 2A = 3{\cdot}500 + 0{\cdot}200 = 3{\cdot}700$$

normal tooth thickness $= \dfrac{\pi}{2\ DP_n}$

$$t_n = \frac{3{\cdot}1416}{2 \times 10} = 0{\cdot}15708$$

To calculate vernier settings virtual pitch diameter D_v

$$= \frac{D}{\cos^2 \phi} = \frac{3{\cdot}500}{0{\cdot}73556} = 4{\cdot}758$$

and virtual number of teeth Z_v

$$= \frac{Z}{\cos^3 \phi} = \frac{30}{0{\cdot}63085} = 47{\cdot}5\text{—say } 48$$

Now $t_n = 0{\cdot}15708$ and

$$\propto \text{ radians} = \frac{t_n}{D_v} = \frac{0{\cdot}15708}{4{\cdot}758} = 0{\cdot}03301$$

one radian $= 57{\cdot}296$ degrees

$\therefore\ 0{\cdot}03301$ radians $= 1^\circ\, 53'\, 27''$

Then $\sin \propto D_v = 0{\cdot}03299 \times 4{\cdot}758$

$t_c = 0{\cdot}1569$

and $f = \dfrac{D_v}{2}$ versine $\propto$

$$= \frac{4{\cdot}758}{2} \times 0{\cdot}00058$$

$f = 0{\cdot}00137$

and $A_c = A + f$

$= 0{\cdot}100 + 0{\cdot}00137$

$= 0{\cdot}1013$

If the pitch line vernier settings for spur gears given in table for 1 DP pitch in Book II are used we proceed as follows :—

$Z_v = 48$ and vernier settings from table for 48 teeth equal

$A_c = 1{\cdot}0123$ and $t_c = 1{\cdot}5706$

Then when $DP_n = 10$ vernier settings equal

$$A_c = \frac{1{\cdot}0123}{10} = 0{\cdot}1012 \text{ and}$$

$$t_c = \frac{1{\cdot}5706}{10} = 0{\cdot}1570$$

To obtain the backlash allowance on each gear (the allowance to be deducted from the normal chordal thickness) we again refer to Book II, from which it is noted that the backlash allowance for 10 DP is 0·004 to 0·008.

Thus $t_c = 0{\cdot}157 \begin{array}{l} -0{\cdot}002 \\ -0{\cdot}004 \end{array}$ measured on the normal plane.

Z_v is equal to 48 teeth therefore the pressure angle can be made either 14½° or 20° pressure angle, but in order to keep the stock of hobs and cutters to a minimum it is recommended that a pressure angle of 20° should be used. Thus the transverse pressure angle will be equal to ψ_t when

$$\tan \psi_t = \frac{\tan \psi_n}{\cos \phi} = \frac{0{\cdot}36397}{0{\cdot}85765} = 0{\cdot}42438$$

$$= 22^\circ\ 59'\ 44''$$

and base circle diameter D_o

$$= D \cos \psi_t = 3{\cdot}500 \times 0{\cdot}92053$$

$$D_o = 3{\cdot}2218''$$

and if the base lead angle is required then

$$\begin{aligned} \cos \text{ base lead angle} &= \cos \psi_n \cos (90^\circ - \phi) \\ &= 0{\cdot}93969 \times 0{\cdot}51429 \\ &= 61^\circ\ 06' \end{aligned}$$

Example No. 2—Spiral Gears.

Spiral gears 15/30 ratio to gear at 3·500 centres, shaft angle 90°, normal pitch of cutter 10 DP.

From layout Fig. 4 make distance OC equal to

$$\frac{Z_1}{DP_n} = \frac{30}{10} = 3{\cdot}000$$

and distance OD is made equal to

$$\frac{Z_2}{DP_n} = \frac{15}{10} = 1{\cdot}500$$

after adjusting line GF the length of which is made equal to

$$\begin{aligned} 2C &= 2 \times 3{\cdot}500 \\ &= 7{\cdot}000 \end{aligned}$$

the distance EF = D pitch circle diameter of wheel
EG = *d* pitch circle diameter of pinion

can be measured off, together with the spiral angles of the wheel and pinion θ and ϕ respectively.

Then from layout $D = 3{\cdot}280$
$d = 3{\cdot}720$
$\theta = 23^\circ\ 40'$
$\phi = 66^\circ\ 20'$

and from formula

$$D = \frac{Z_1}{DP_n \cos\theta} = \frac{30}{10 \times 0{\cdot}91590} = 3{\cdot}275$$

$$d = \frac{Z_2}{DP_n \cos\phi} = \frac{15}{10 \times 0{\cdot}40141} = 3{\cdot}736$$

$$\text{Then centres } \frac{3{\cdot}275 + 3{\cdot}736}{2} = 3{\cdot}5055$$

which means that the gears would be 0·0055 wide of centres, and an adjustment must be made to the spiral angle.

In this case the cosine of the largest angle must be increased slightly, and a new value obtained for D and *d*, the adjustments to be continued until D and *d* divided by two equals the centre distance within 0·001.

By trial and error the following values conform to the requirement :—

$$D = \frac{30}{10 \times 0{\cdot}91531} = 3{\cdot}277 \text{ when } \cos 0{\cdot}91531 = 23^\circ\ 45'$$

$$\text{and } d = \frac{15}{10 \times 0{\cdot}40275} = 3{\cdot}724 \text{ when } \cos 0{\cdot}40275 = 66^\circ\ 15'$$

$$\text{from which } C = \frac{3{\cdot}277 + 3{\cdot}724}{2} = 3{\cdot}5005$$

which will be satisfactory, and the gear having the largest spiral angle will be the driver and referring to Fig. 5, and after fixing the hand of the gears, the direction of thrust can be obtained from the figures.

When the above values have been finalised as described, the method used for calculating the other particulars required for cutting the gears are exactly as those used for the helical gears previously given.

CHAPTER II.

FORM MILLING.

Since the introduction of generating machines for the production of spiral and helical gears, the practice of cutting these gears by the form milling method has practically fallen out of use, but it is still used when replacement gears are required for machine tools, or for small lot production.

The machine used is a standard universal milling machine which is suitable for cutting gears up to a maximum spiral angle of 50°, this depending, of course, on the maximum angular movement of the machine table, the table being set to agree with the spiral angle of the gear given on the drawing.

For spiral angles from zero to practically 90° a plain milling machine can be used, but in this case a spiral milling attachment or universal milling head is also necessary to obtain the required angular movement of the cutter, due to the fact that the table on a plain milling machine does not swivel.

In the case of the universal machine the rotary form cutter is mounted on the cutter arbor of the machine, and set central with the gear blank by using spacing collars or adjusting the transverse slide in the ordinary way, whereas if a plain milling machine is used the cutter is mounted on the arbor of the attachment, and to set the cutter central with the gear blank the adjustment has generally to be made on the transverse slide only, after which the saddle should be locked.

The gear blank is mounted on a mandrel which is held between the centres of a dividing lead and tailstock, mounted on the table of the machine, and suitable gearing is arranged to drive the spindle of the dividing head through the driving worm from the lead screw of the machine. Therefore, when the longitudinal traverse is engaged the gear blank is fed past the cutter, and at the same time rotated by the gearing from the lead screw, the speed of rotation being arranged according to the lead of spiral to be cut.

The power feed can be used during the cutting operation for all leads between 2 inches and 60 inches, but for leads less than 2 inches it is advisable to feed the table by hand using the crank handle of the dividing head due to the high ratio of gearing required for the small leads.

In Fig. 7 is shown an arrangement of the gearing from the milling machine lead screw to the driving shaft of the dividing head usually named "gear on worm," while the driving gear on the lead screw is named "gear on screw," intermediate gears being used to connect the two shafts, these gears being mounted on a quadrant swung from the "gear on worm" shaft.

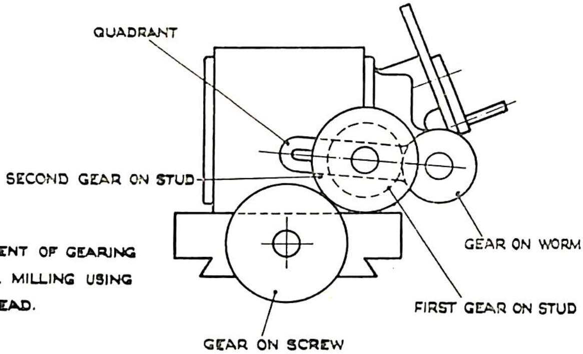


FIG. 7. ARRANGEMENT OF GEARING FOR SPIRAL MILLING USING DIVIDING HEAD.

If gears with an equal number of teeth were to be placed on the table feed screw and the dividing head worm shaft, then the lead of the gear being cut would be the distance the milling machine table will traverse while the gear blank makes one complete revolution.

As previously explained in Chapter I, the lead of spiral is the distance measured along the axis of the gear, in which the spiral makes one full turn around the gear on the pitch cylinder. Therefore, the lead of the milling machine can be expressed as the lead of spiral that will be cut on a gear blank when gears with an equal number of teeth are placed on the table feed screw, and the dividing head worm shaft, with an idler gear or gears of suitable size placed between the driving and driven gears.

To obtain the lead of any milling machine we proceed as follows :

Let V = number of teeth in dividing head worm wheel.
G = number of threads in dividing head worm.
S = lead of milling machine table feed screw.
L = lead of milling machine.

Then $L = V \times G \times S$.

Most milling machines are manufactured with a table feed screw of 0·250 lead while the majority of dividing heads have 40 teeth cut in the dividing head wormwheel and this is driven by a single thread worm.

Then the lead of a milling machine having these characteristics when L = lead of machine would be

$$40 \times 1 \times 0{\cdot}250 = 10 \text{ inches,}$$

from which it is correct to assume that a milling machine having equal gearing from the table feed screw to the dividing head would cut a lead of 10 inches when milling the teeth of a spiral or helical gear.

The gear train necessary for any spiral can be calculated from the following formula :—

$$\frac{\text{lead of machine}}{\text{lead of spiral to be cut}} = \frac{\text{drivers}}{\text{driven}}$$

when "Gear on screw" is the driving gear and the "gear on worm" is the driven gear or expressed as a formula :—

$$\frac{\text{drivers}}{\text{driven}} = \frac{\text{gear on screw} \times \text{first gear on stud}}{\text{second gear on stud} \times \text{gear on worm}}$$

The change gears supplied with the dividing head and listed in Book II, Chapter 2, are usually sufficient to obtain practically any lead between 0·670 to 60 inches, and the actual leads obtained are listed in the instruction book supplied with the dividing head. Any leads required which are not listed may, of course, require special change gears, the numbers of teeth in these gears being calculated in accordance with the formula given above.

Example.

Change gears are required to cut a lead of 2·500 on a milling machine 10 inches lead,

$$\text{Then } \frac{\text{drivers}}{\text{driven}} = \frac{10{\cdot}000}{2{\cdot}500} = \frac{56 \times 48}{28 \times 24}$$

and 56 teeth = gear on screw.
48 teeth = first gear on stud.
28 teeth = second gear on stud.
24 teeth = gear on worm.

Selecting the Cutter.

To select the correct form milling cutter for cutting a spiral or helical gear of standard proportions, the list of eight cutters given in Book II for spur gears is referred to.

When the cutters are used for spiral or helical gears, however, the *virtual* number of teeth in the gear is used to obtain the correct cutter number, and not the *actual* number of teeth.

For example, to select the cutter number for cutting a spiral gear 30 teeth 45° spiral angle we proceed as follows :—

Let Z = actual number of teeth.
Z_v = virtual number of teeth.
ϕ = spiral angle.

$$\text{Then } Z_v = \frac{Z}{\cos^3 \phi} = \frac{30}{\cos^3 45^\circ} = 84{\cdot}89, \text{ say } 85 \text{ teeth,}$$

and from table a No. 2 cutter will be required.

The standard pressure angle on the normal plane of gears cut with these cutters is 14½° and the tooth proportions are in accordance with the basic rack for the 14½° composite system as shown in Fig. 6, Book II.

For other pressure angles which may be used or for gears of special proportions having a large spiral angle, the tooth shape on the normal section should be obtained from the involute form on the transverse plane, and the method used will be described below.

To obtain the tooth space shape on the normal section for the purpose of producing a suitable form milling cutter the shape can be obtained either by graphical methods or by calculation, involving the use of involute functions.

If the graphical method is used the tooth space shape is first drawn out as shown in Fig. 8 in the same manner as was used for a spur gear, the pressure angle on this plane being made equal to ψ_t.

Lines AA and BB are then drawn above the actual pitch circle diameter D, indicated by CC in figure, and lines DD and EE are drawn below line CC, the distances between the lines being dependent upon the size of tooth under consideration.

The virtual pitch circle diameter D_v is then drawn as shown in Fig. 9, and the normal circular width of the tooth space *cc* stepped off each side the centre line this distance being made equal to :—

CC cos ϕ when CC = tooth space width on transverse plane.
ϕ = spiral angle of gear on pitch circle diameter D.

To obtain the distances *aa, bb, dd* and *ee,* it is first necessary to calculate the various spiral angles on the respective diameters, from the formula :—

$$\tan \phi_a = \frac{(D + 4f)\ \pi}{L}$$

$$\tan \phi_b = \frac{(D + 2f)\ \pi}{L}$$

$$\tan \phi_d = \frac{(D - 2f)\ \pi}{L}$$

$$\tan \phi_e = \frac{(D - 4f)\ \pi}{L}$$

When ϕ_a = spiral angle on line AA
ϕ_b = ,, ,, ,, ,, BB
ϕ_d = ,, ,, ,, ,, DD
ϕ_e = ,, ,, ,, ,, EE
When D = actual pitch circle diameter, line CC
f = radial distance between each circle
L = lead of spiral

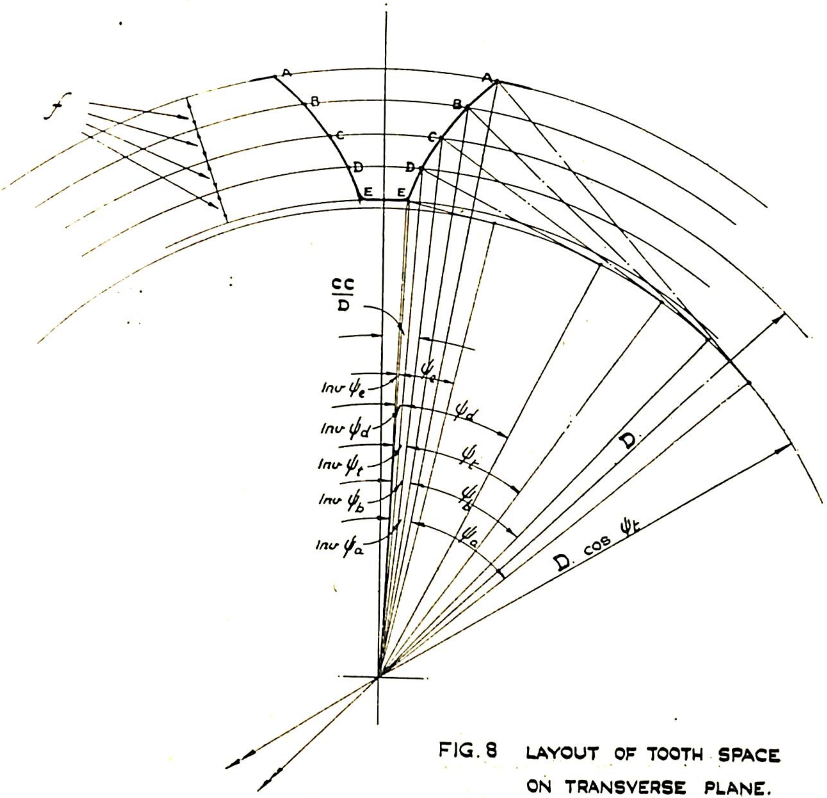


FIG. 8 LAYOUT OF TOOTH SPACE ON TRANSVERSE PLANE.

The distance AA, Fig. 8, is then multiplied by cosine ϕ_a to obtain distance "*aa*", Fig. 9, the same procedure being used to obtain the distances *bb, dd* and *ee* on the normal plane, Fig. 9, using the corresponding spiral angles, the points obtained being joined together with a "curve" the distances "*f*" being the same as were used for Fig. 8.

More points can be used than are actually shown in the figures but again this depends upon the size of tooth under review.

To calculate the distances AA, BB, DD and EE, Fig. 8, involute functions have to be used, and from the value CC, and the various diameters D_a, D_b, D, D_d, D_e, it is a simple matter to obtain the distances required on the transverse plane, after which the same procedure is adopted as is used for the graphical method to obtain the co-ordinates on the normal plane, Fig. 9.

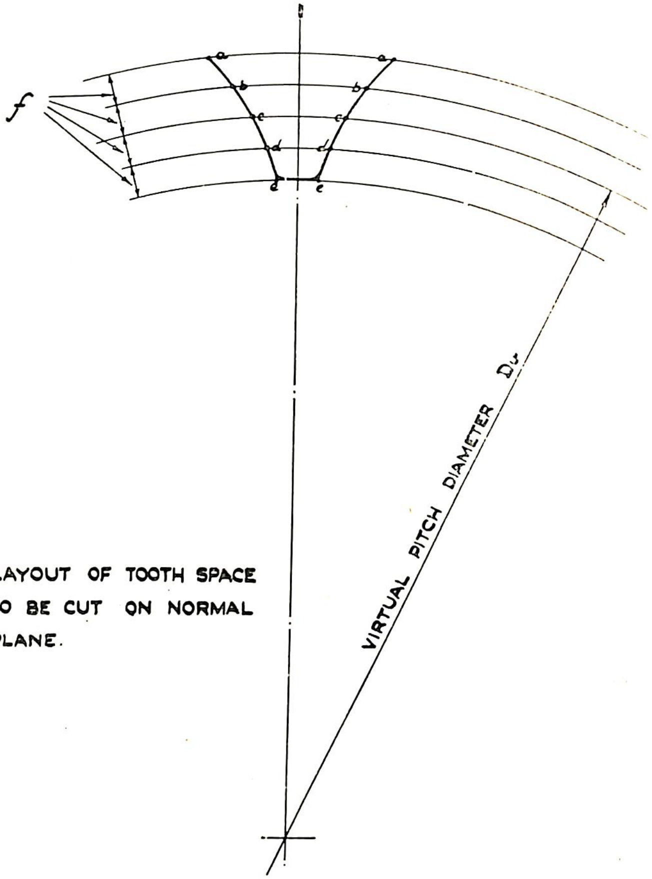


FIG. 9. LAYOUT OF TOOTH SPACE TO BE CUT ON NORMAL PLANE.

Then $D_a = D + 4f$

$D_b = D + 2f$

$D_d = D - 2f$

$D_e = D - 4f$

and
$$AA = D_a \left[\left(\frac{CC}{D} - \text{inv. } \psi_t + \text{inv. } \psi_a\right)\right]$$

$$BB = D_b \left[\left(\frac{CC}{D} - \text{inv. } \psi_t + \text{inv. } \psi_b\right)\right]$$

$$DD = D_d \left[\left(\frac{CC}{D} - \text{inv. } \psi_t + \text{inv. } \psi_d\right)\right]$$

$$EE = D_e \left[\left(\frac{CC}{D} - \text{inv. } \psi_t + \text{inv. } \psi_e\right)\right]$$

care to be taken that D_d or D_e is never less than the base circle diameter D_o when

ψ_t = transverse pressure angle on D or CC

ψ_a = transverse pressure angle on D_a

ψ_b = transverse pressure angle on D_b

ψ_d = transverse pressure angle on D_d

ψ_e = transverse pressure angle on D_e

When
$$\cos \psi_a = \frac{D \cos \psi_t}{D_a}$$

$$\cos \psi_b = \frac{D \cos \psi_t}{D_b}$$

$$\cos \psi_d = \frac{D \cos \psi_t}{D_d}$$

$$\cos \psi_e = \frac{D \cos \psi_t}{D_e}$$

Indexing.

When cutting spiral or helical gears using the dividing head the *actual* number of teeth should be indexed for each tooth by using the simple indexing method only.

Differential indexing cannot be employed as the change gears are used for obtaining the lead of spiral, and this point should be borne in mind when deciding upon the actual number of teeth in the gear, and prime numbers should be avoided.

It is essential therefore that the divisions are obtained only by the use of the index plates supplied with the dividing head, the number of holes in the plates usually supplied being as follows :—

15, 16, 17, 18, 19, 20, 21, 23, 27, 29, 31, 33, 37, 39, 41, 43, 47 & 49.

Then to calculate the number of turns and the number of holes, the crank lever on the dividing head must be turned when using simple indexing, we proceed as follows :—

Let N = *actual* number of teeth to be cut.
V = number of teeth in dividing head wormwheel.
G = number of threads in dividing head worm.
X = number of revolutions of crank lever to perform one division.

Then $X = \frac{V}{N}$

or in words :—

To obtain a given number of divisions, divide the number of turns required for one revolution of the dividing head spindle, by the number of divisions into which the periphery of the work is to be divided, for example :—

To cut a gear having 30 teeth we get

$$X = \frac{40}{30} = 1\tfrac{1}{3} \text{ revolutions of crank lever,}$$

thus the crank lever must be moved one full revolution and 11 holes using a 33 index circle.

CHAPTER III.

HOBBING.

Spiral and helical gears can usually be produced by the hobbing method on the same machine as used for hobbing spur gears, the limiting factor being the spiral angle.

A machine primarily designed for hobbing spur gears is usually capable of producing helical gears up to a maximum spiral angle of 35°.

For cutting spiral gears having a spiral angle greater than 30° it is recommended that a universal hobbing machine be used, *i.e.*, one capable of cutting spur, spiral, and worm gears by the hobbing method, and the machine should also be fitted with differential change gears.

When hobbing spiral or helical gears an auxiliary rotation is necessary corresponding to the angle of the tooth spiral, and if a machine without a differential is being used, the calculation of the index change gear train often necessitates lengthy and tedious calculations, due to the fact that this gear train alone governs the pitch and auxiliary movement.

In addition, should the feed be altered a new calculation is necessary, while after the hob saddle has been run back after completing say the first cut the hob stands in no definite relation to the gear teeth, and must, therefore, be set again.

The formula used also contains the number of teeth, and different gear trains are again necessary for a gear and pinion having different numbers of teeth, therefore as two entirely different trains of gears will be required, they may not in every case reproduce the accuracy of tooth spiral required.

Machines of this type should therefore only be used when a very large number of the same part is required, and where only one cut is necessary to complete the tooth cutting operation.

For general production, machines fitted with a differential change gear train are ideal, as this is entirely independent of the feed and the number of teeth.

It is also possible to wind back the hob saddle after the first cut, re-engage the feed for the second cut, and find that the hob tooth is in a positive relative position to the gear tooth, while, in addition, it is possible to change the feed for the finishing cut as this does not affect the differential change gears.

In either case the gears are calculated according to the instructions given in the handbook supplied with the machines and as, of course, the formula may vary with each type of machine, it is therefore not possible to quote formulas to cover all makes of machines.

Special care should be taken when designing spiral gears to ensure that the clearances for the hob run out are sufficient, especially when gears with shoulders are under consideration.

This clearance will, as in the case of spur gears, depend on the outside diameter of the hob being used, but additional clearance is required for spiral and helical gears, due to the hob swivel slide being swung over, normal to the spiral angle plus or minus the helix angle of the hob.

To determine the minimum distance between the gear face and the shoulder, a layout should be made similar to that shown in Fig. 10, and to prepare this layout the outside diameter and helix angle of the hob should be known, together with the spiral angle, and tooth depth of the gear to be cut.

Hand of Hob.

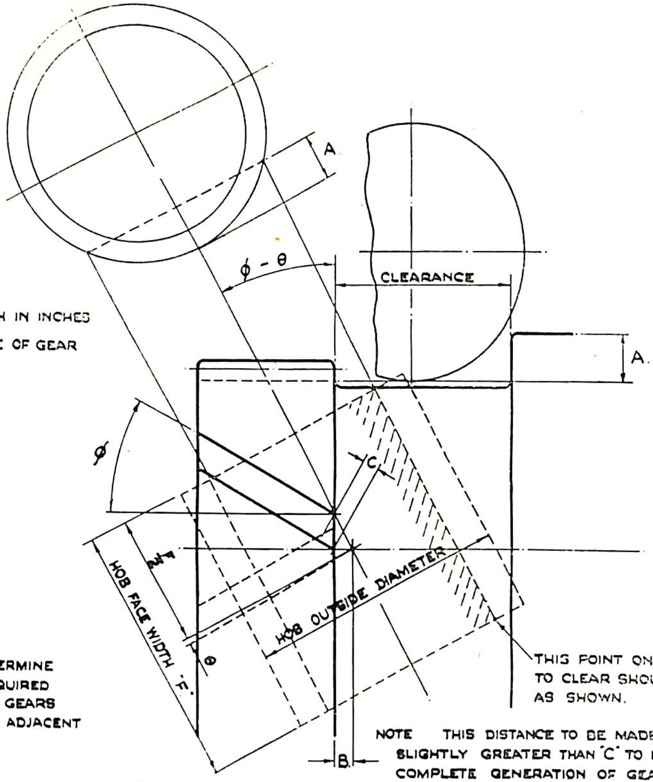
After mounting the hob on the arbor the hob swivel slide should be rotated so that the normal plane of the gear and hob coincide, and the direction in which the slide is rotated for either right or left hand gears is shown in Fig. 11.

$C = p_n \tan \phi$

WHEN p_n = NORMAL PITCH IN INCHES

ϕ = SPIRAL ANGLE OF GEAR

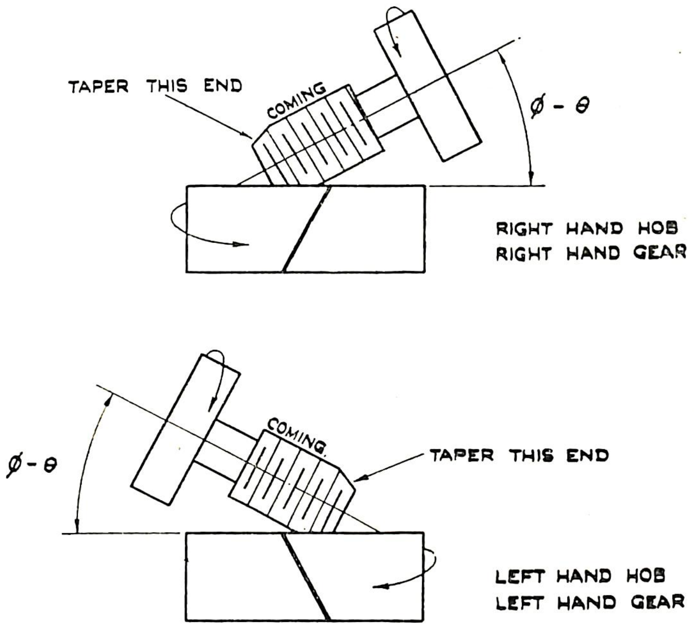
THEN $B = C \sec \phi$



THIS POINT ON HOB TO CLEAR SHOULDER AS SHOWN.

NOTE THIS DISTANCE TO BE MADE SLIGHTLY GREATER THAN "C" TO ENSURE COMPLETE GENERATION OF GEAR TEETH.

FIG. 10. LAYOUT TO DETERMINE CLEARANCE REQUIRED WHEN HOBBING GEARS WITH SHOULDERS ADJACENT TO TEETH.



ϕ = SPIRAL ANGLE OF GEAR.
θ = HELIX ANGLE OF HOB.

FIG. 11. SHEWING POSITION OF HOB SWIVEL SLIDE IN RELATION TO SPIRAL GEAR.

The hand of the hob used for cutting spiral gears, even if the spiral angle is smaller than 20° is important and it is recommended that a right-hand hob should be used for cutting a right-hand gear, and a left-hand hob for a left-hand gear.

This rule ensures that the torque due to cutting pressure operates in the opposite direction to the rotation of the work table, thus avoiding any danger of vibration and chatter detrimental to the finish of the tooth flanks.

The same hob as used for spur gears can be used for spiral or helical gears providing that the normal pitch and pressure angle of the hob is the same as the gear on the normal plane, but for spiral gears above 20° spiral angle it is recommended that the hob should be tapered at the leading end, the amount of taper being about 20°, the taper commencing at the root of the hob tooth, and the position of the taper in relation to the rotation and hand of hob also being shown in Fig. 11.

The setting angle of the hob swivel slide is made equal to $\phi - \theta$ when

ϕ = spiral angle of gear.
θ = helix angle of hob

when using a right-hand hob for a right-hand gear, or a left-hand hob for a left-hand gear.

Should, however, a right-hand hob be used for cutting a left-hand gear, or a left-hand hob for cutting a right-hand gear, the hob swivel slide setting angle is made equal to $\phi + \theta$ while the hob runs in the same direction, but the direction of the work spindle is reversed to that shown in figure.

Index Gears.

The information regarding the index gears necessary for obtaining the required number of teeth in the gear is given in the instruction book supplied with the machine, and in this case the *actual* number of teeth in the gear to be cut is used, to obtain the correct train of gears.

Feed.

Another train of gears is necessary to obtain the desired feed, and, as previously explained, for hobbing machines not fitted with a differential, the feed is bound up with the index gears, to obtain the auxiliary movement necessary for tooth spiral.

The same speeds can be used as those specified for hobbing spur gears, but the feeds should be slightly less, being made equal to the feed used for spur gears, multiplied by the cosine of the spiral angle.

CHAPTER IV.

RACK PLANING.

As an alternative to hobbing, the rack planing method is most commonly used for generating spiral and helical gears.

The same machine as used for spur gears can, in the majority of cases, be utilised for both types providing that the cutter slide can be swivelled to an angle corresponding to the spiral angle of the gear being cut.

As, however, the lead of the gear being cut is not used in the setting up of the machine, it is important that the cutter slide angle is set as accurately as possible.

There is another slight difference in the set up of the machine, namely, in the calculation of the change gear train to give the correct amount of pitch travel to the cutter after each index.

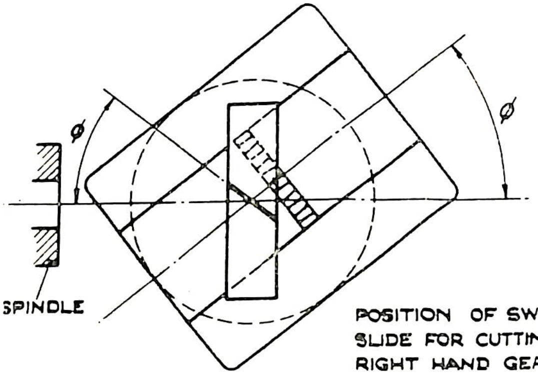
In the calculations, the transverse pitch value must be used, and not the normal pitch which, is, of course, the pitch of the cutter, and given the normal pitch DP_n the transverse pitch DP_t in terms of diametral pitch will be equal to $\frac{DP_n}{\sec \phi}$ when ϕ = spiral angle of gear.

A longer stroke of the cutter is required for cutting spirals than for spur gears of equal face width, the stroke being of such length to clear the blank at each end of the stroke, and this point should be closely watched when deciding upon the size of machine suitable for cutting the teeth of the gear under consideration.

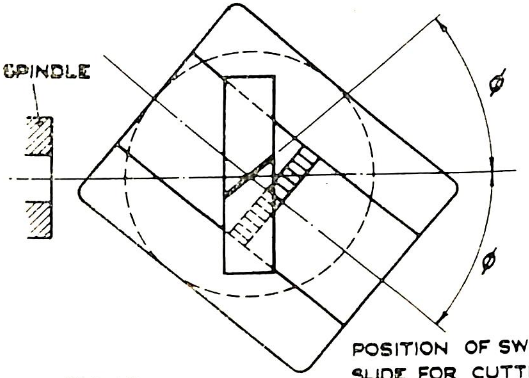
It is therefore recommended that layouts giving the specifications of machines installed are made, to assist in this matter, and any peculiarities of the machine recorded.

The direction in which the cutter slide must be swivelled in relation to the hand of spiral is shown in Fig. 12, but with regard to the direction of cut this varies on different makes of machines and reference should be made to the handbook supplied.

The cutter used for spiral or helical gears is the same as that used for a spur gear of the same pitch as the *normal pitch* of the spiral gear, and it is for this reason that these gears are made of standard proportions on the normal plane, except, of course, gears designed to be cut by the gear shaper method which is dealt with in Chapter V.



POSITION OF SWIVEL SLIDE FOR CUTTING RIGHT HAND GEAR.



POSITION OF SWIVEL SLIDE FOR CUTTING LEFT HAND GEAR

FIG. 12.

Indexing.

The index change gears are selected to cut the *actual* number of teeth in the gear, the formula varying with different makes of machines, but the formula can be obtained from the handbook supplied, and recorded on the machine layout.

Gears with shoulders should be closely watched and it is advisable to prepare a layout showing the cutter in relation to the gear, before deciding upon the minimum distance which can be allowed between the face of gear and the shoulder. Such a layout is shown in Fig. 13, and before preparing the layout the length and section of the cutter must be known.

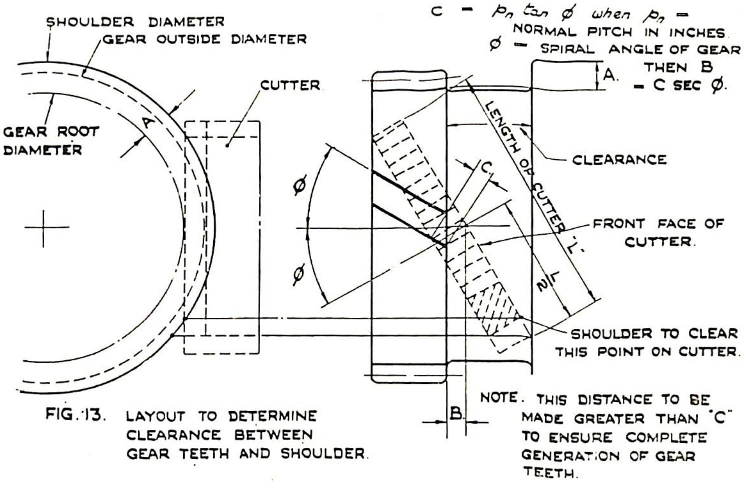

SHOULDER DIAMETER
GEAR OUTSIDE DIAMETER
GEAR ROOT DIAMETER
A
CUTTER
ϕ
ϕ
C = p_n tan ϕ when p_n = NORMAL PITCH IN INCHES.
ϕ = SPIRAL ANGLE OF GEAR
A.
THEN B = C SEC ϕ.
LENGTH OF CUTTER "L".
C.
CLEARANCE
FRONT FACE OF CUTTER.
L/2
SHOULDER TO CLEAR THIS POINT ON CUTTER.
B.
NOTE. THIS DISTANCE TO BE MADE GREATER THAN "C" TO ENSURE COMPLETE GENERATION OF GEAR TEETH.

FIG. 13. LAYOUT TO DETERMINE CLEARANCE BETWEEN GEAR TEETH AND SHOULDER.

The only difficulty in the setting up of the machine may be found in obtaining a gear train to give the correct amount of pitch travel after each index, as it will be found that the gear ratio obtained is usually a decimal ratio.

Various methods of obtaining suitable gears for a given ratio are in use, namely, continued fractions, Mold's ratios, Brocots tables or logarithms of gear ratios, all of which will be suitable for obtaining change gears to give the accuracy required, although it may in some cases be necessary to make special change gears.

CHAPTER V.

GEAR SHAPING.

Helical gears can be cut by the gear shaper method on a gear shaping machine fitted with a helical guide assembly, which takes the place of the straight guide, used when cutting spur gears.

The straight guide can be removed as a unit and replaced by the helical guide assembly which is used to produce the tooth spiral required, this assembly consisting of three principal parts, viz. :—

1. Taper gib and shoe, the shoe being split into two pieces.
2. A helical guide which is attached to the cutter spindle.
3. A casing or sleeve.

The two piece shoe is secured in the casing by cap screws, and adjusted by means of the taper gib, while the guide attached to the cutter spindle slides along the two inclined faces of the shoe at each stroke of the cutter spindle.

Due to the fact that the shoe and casing are stationary, and the guide is attached to the cutter spindle, on each stroke of the spindle a rotary motion is imparted to the cutter, thus producing the tooth spiral on the gear blank being cut.

Owing to the construction of the guide assembly the gear shaper method is not recommended for spiral angles greater than 35°, and for this reason can only be used for helical gears.

In order to keep the number of guides as low as possible, two standard helical guides have been introduced by the Fellows Gear Shaper Co., and these give approximate spiral angles of 15° and 23° respectively.

The exact leads of the two guides are 41·270 inches for 15° spiral angle, and 25,904 inches for 23° spiral angle based on a cutter pitch circle diameter of 3·500 inches, right and left-hand guides being required for each lead.

Therefore as it is not always possible to design a cutter exactly 3·500 pitch circle diameter (as is the case for say a 7 DP cutter) the spiral angle of the cutter would be slightly greater than the 15° or 23° mentioned above, and this would also apply to the angle of the gear being cut.

The hand of the cutter is always opposite to the hand of the gear to be cut while the hand of the guide must be the same as the cutter, *i.e.*, to cut a right-hand gear, a left-hand cutter, and a left-hand guide will be required.

Given the lead of the guide, to obtain the spiral angle of the cutter and gear we proceed as follows :—

DP_t = diametral pitch on *transverse* plane.
D_c = pitch circle diameter of cutter.
Z_c = number of teeth in cutter.
L_c = lead of cutter and guide.
ϕ_c = spiral angle of cutter and gear.

Then $Z_c = D_c \times DP_t$

which will be a whole number when

D_c = 3·500 inches and
DP_t = 4, 6, 8, 10 and 12

should DP_t equal 5, 7 or 9, then Z_c will not be a whole number when D_c is equal to 3·500 inches, therefore the nearest greater whole number should be used, and the pitch circle diameter obtained from the formula.

$$D_c = \frac{Z_c}{DP_t}$$

After finalising the values D_c and Z_c, the value ϕ_c can be obtained as follows :—

$$\tan \phi_c = \frac{D_c \pi}{L_c}$$

The lead of the gear can then be calculated from the formula

$$L = \frac{D \pi}{\tan \phi_c}$$

When L = lead of spiral of gear.
D = pitch circle diameter of gear.

It will be found that the two leads mentioned above will cover a large range of work, and as the spiral angle is first governed by the transverse pitch and width of face, the face width can be made of such a width to give continuous action after fixing the transverse pitch and spiral angle.

Therefore to obtain the minimum width of face let

F = face width of gear.
ϕ_c = spiral angle of cutter.
DP_t = diametral pitch on transverse plane.

$$\text{Then } F = \frac{\cot \phi_c \, \pi}{DP_t}$$

to which should be added at least 5% of the value to ensure continuous contact is maintained.

It should be specially noted that the diametral pitch DP_t is on the transverse plane, and not on the normal plane as was the case for helical gears cut by the hobbing and rack planing methods, and for this reason the stub tooth has been adopted for helical gears cut by the gear shaper method.

For example a helical gear 5 DP pitch on the transverse plane is made with an addendum and dedendum of 7 DP proportions while a gear of 6 DP pitch on the transverse plane is made with an addendum and dedendum of 8 DP proportions, and in Fig. 14 a table has been prepared giving the necessary particulars for each pitch which can be cut by the gear shaper method.

Should it be required to cut a gear of any other spiral angle within the range of the machine, it would, of course, be necessary to provide additional guides and cutters to conform to the spiral angle required.

An example is given below for a helical gear 40 tooth 5/7 DP to be cut on a helical gear shaper using a guide of 41·270 inches.

Then $Z_c = 3{\cdot}500 \times 5 = 17{\cdot}5$

Let $Z_c = 18$

and $D_c = \dfrac{18}{5} = 3{\cdot}600$

$$\tan \phi_c = \frac{3{\cdot}600 \times \pi}{41{\cdot}270} = 0{\cdot}27404 = 15^\circ\ 20'$$

Lead of gear L

$$= \frac{8{\cdot}000 \times \pi}{0{\cdot}27404} = 91{\cdot}71 \text{ inches}$$

Then minimum width of face F to give continuous action

$$= \frac{3{\cdot}6470 \times \pi}{5} = 2{\cdot}291$$

Make 2·312 to 2·375 inches to ensure continuous contact.

The normal pitch in terms of the diametral pitch denoted by DP_n can then be calculated as follows :—

$$DP_n = DP_t \sec \phi_c$$

$$\text{or } p_n = \frac{\pi}{DP_t} \cos \phi_c$$

when p_n = pitch on normal plane in inches, from which the vernier settings can be calculated as shown in Chapter I.

PROPORTIONS OF HELICAL GEARS CUT BY THE GEAR SHAPER METHOD

TRANSVERSE DIAMETRAL PITCH 'DP_t'	SPIRAL ANGLE OF GEAR AND CUTTER 'ϕ_c'	LEAD OF CUTTER IN INCHES 'L_c'	MINIMUM WIDTH OF FACE 'F' ✱	ADDENDUM OF GEAR	DEDENDUM OF GEAR	PITCH CIRCLE DIAMETER OF CUTTER 'D_c'	NUMBER OF TEETH IN CUTTER 'Z_c'	PITCH ON NORMAL PLANE IN INCHES 'p_n'
$\frac{5}{7}$	15° – 20′	41·270	2·291	0·1429	0·1786	3·600	18.	0·6059
$\frac{5}{7}$	23° – 35′	25·904	1·439	0·1429	0·1786	3·600	18	0·5758
$\frac{6}{8}$	14° – 55′	41·270	1·965	0·1250	0·1563	3·500	21	0·5059
$\frac{6}{8}$	23° – 00′	25·904	1·233	0·1250	0·1563	3·500	21	0·4819
$\frac{7}{9}$	15° – 12′	41·270	1·651	0·1111	0·1389	3·571	25	0·4331
$\frac{7}{9}$	23° – 25′	25·904	1·036	0·1111	0·1389	3·571	25	0·4118
$\frac{8}{10}$	14° – 55′	41·270	1·474	0·1000	0·1250	3·500	28	0·3794
$\frac{8}{10}$	23° – 00′	25·904	0·925	0·1000	0·1250	3·500	28	0·3615
$\frac{9}{11}$	15° – 9′	41·270	1·289	0·0909	0·1136	3·555	32	0·3369
$\frac{9}{11}$	23° – 20′	25·904	0·809	0·0909	0·1136	3·555	32	0·3205
$\frac{10}{12}$	14° – 55′	41·270	1·179	0·0833	0·1042	3·500	35	0·3035
$\frac{10}{12}$	23° – 00′	25·904	0·740	0·0833	0·1042	3·500	35	0·2892
$\frac{12}{14}$	14° – 55′	41·270	0·982	0·0714	0·0893	3·500	42	0·2529
$\frac{12}{14}$	23° – 00′	25·904	0·616	0·0714	0·0893	3·500	42	0·2409

✱ TO VALUE 'F' SHOULD BE ADDED AT LEAST 5% OF THE VALUE TO ENSURE CONTINUOUS CONTACT.

FIG 14.

Tooth Proportions.

The pressure angle of the stub tooth system is standardised at 20° on the transverse plane for helical gears cut by the gear shaping process, and the tooth form on this plane is exactly the same as a spur gear of the same tooth proportions.

This point is important as it means that a pair of spur gears can be replaced by a pair of helical gears cut by this method, if so desired.

CHAPTER VI.

PROFILE GRINDING.

The grinding of helical gear tooth profiles on a production basis is at present carried out on either Maag or Lees-Bradner machines using the generating method.

It is understood, however, that the form wheel principle is being applied in some cases but due to the difficulty of obtaining the tooth shape on the normal plane by this method, as explained in Chapter II, it is considered that the generating method is the only one which will produce the correct theoretical shape on the normal plane, especially when used for helical gears of large spiral angle.

The generation takes place on the transverse plane, and the grinding wheel is arranged to represent the sides or flanks of a rack cutter in exactly the same way as for the rack planing method described in Chapter IV.

That is, the gear rolls on its transverse plane and the rack form of the grinding wheel is swung round to the normal plane of the gear being cut, *i.e.*, to an angle equal to the spiral angle of the gear.

On the Lees-Bradner machine master racks and master gears are used to obtain the generating motion, on the transverse plane, and they are also used to index the gear from tooth to tooth after each pass of the grinding wheel.

The racks, of which there are three in number, are arranged to mesh with the master gear, and the face width of the gear is made equal to the total face width of the three racks.

The tooth proportions of the racks and master gear are not vitally important, providing that the axial pitch of the racks and the transverse pitch of the master gear are made the same as the transverse pitch of the helical gear to be ground, while the number of teeth in the master gear should also be the same as the actual number of teeth in the helical gear.

The racks and master gear are not made as helical gears, however, but as ordinary spur gears, the spiral angle being obtained from the swivel movement of the grinding wheel head previously mentioned, and the inclination of the grinding wheel spindle is made equal to the normal pressure angle of the gear to be ground, and denoted by the symbol ψ_n in Fig. 4, Book I.

Therefore, if for instance the grinding wheel head was to be set at zero a spur gear could be ground, the transverse pitch of which would be equal to the transverse pitch of the master gear and racks being used, the pressure angle for spur gears being the same as the inclination angle of the grinding wheel spindle from the horizontal.

Full details of the three racks and master gear are given with the machine specification, or in the handbook supplied, care, however, being taken to ensure that the racks and master gear are accurately manufactured and ground on the tooth profiles.

The length of the racks is also important and racks designed for spur gears of the same transverse pitch should be made longer when required for helical gears by an amount equal to F tan ϕ.

When F = face width of gear to be ground.
ϕ = spiral angle of gear.

and in Fig. 15 is shown a method used to obtain the minimum length of racks for both spur and helical gears to obtain full generation.

The tooth shape shown in figure is a view of the helical gear on the transverse plane, the pressure angle of the gear on this plane being made equal to ψ_t.

The outside diameter, pitch circle diameter, root diameter and clearance circles are first drawn, and in conjunction with the pressure angle the base circle diameter is indicated.

Line EG is then drawn tangential to the working depth circle the length of this line being determined by the base circle diameter D_o. Line EH is drawn at an angle ψ_t to line EG and this line extends to the outside diameter circle J after which it is only necessary to complete the triangle EGH, making line GH 90° to line EH.

The minimum length of rack for a spur gear is then equal to the length of line EG but for a helical gear the length is indicated in the plan view of the figure by projecting the line EG.

On this view the width of face F is indicated together with the spiral angle of the gear and the parallologram EGKL completed as shown, the minimum length of rack being made equal to the projected length of EL as shown.

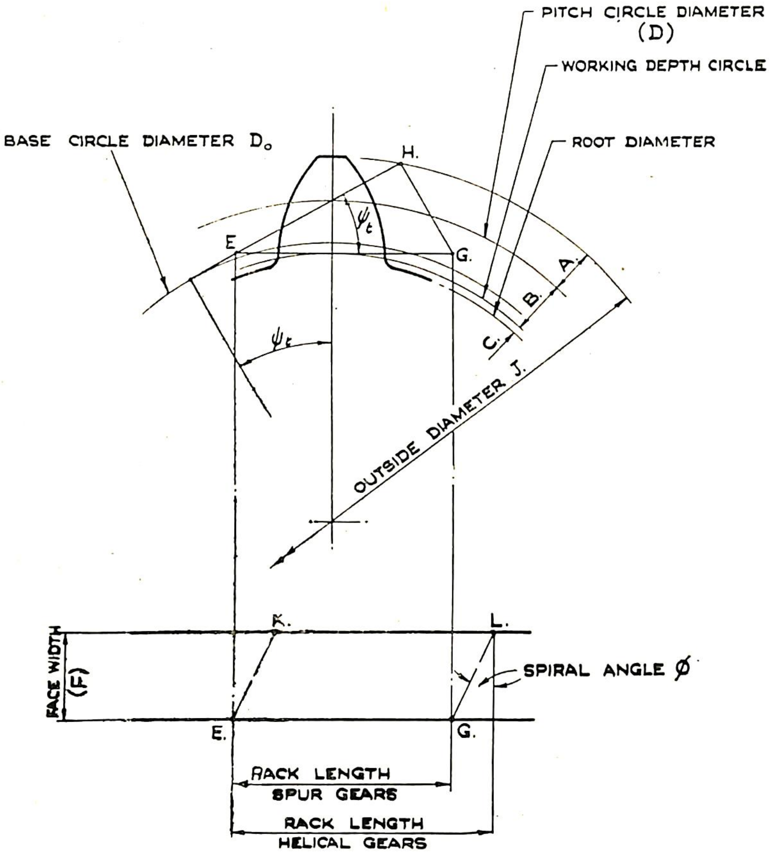


FIG. 15. LAYOUT TO DETERMINE MINIMUM LENGTH OF RACK FOR SPUR AND HELICAL GEARS TO OBTAIN FULL GENERATION

When A = addendum of gear.
B = dedendum of gear less clearance.
C = clearance.

For helical gears ground on a Maag machine two grinding wheels are used as was the case when grinding spur gears, Book II, page 44, but provision is made for the grinding wheel heads to be swung normal to the tooth spiral.

Master racks and gears are not used, as was the case for the Lees-Bradner machine, but in their place a pitch block and tapes are fitted to generate the tooth form, while an index plate is used to index the gear from tooth to tooth after each traverse of the slide.

The diameter of this pitch block is calculated, taking into consideration the inclination of the grinding wheel spindles, and the spiral angle of the gear to be ground, otherwise the dimensions of the pitch blocks are the same as were used for spur gears.

The formula used, however, to calculate the pitch block diameter for a helical gear is rather more complicated than that used for spur gears, and due to the inclination of the grinding wheel heads the gear is actually ground at a slightly different spiral angle to that given on the drawing which is the angle of spiral on the pitch cylinder diameter D.

To calculate the diameter of the pitch block for grinding a helical gear we proceed as follows :—

Let ψ_t = transverse pressure angle.
ϕ = spiral angle.
ψ_n = pressure angle on normal plane.
θ = spiral angle used for setting grinding wheel head termed "grinding angle."
ψ_G = pressure angle of generation on transverse plane.
D = pitch diameter of gear.
D_o = base diameter of gear.
D_G = pitch diameter on which generation takes place.
D_p = diameter of pitch block.
y = thickness of tapes.

Given ψ_n and ϕ

$$\tan \psi_t = \frac{\tan \psi_n}{\cos \phi}$$

Then $D_o = \cos \psi_t \, D$

$$\sin \theta = \sin \phi \, \frac{\cos \psi_n}{\cos 15^\circ}$$

When 15° = inclination angle of grinding wheel spindle on the Maag machine.

$$\tan \psi_G = \frac{\tan 15^\circ}{\cos \theta}$$

$$D_G = D_o \sec \psi_G$$

and diameter of pitch block

$D_P = D_G - y$

when $y = 0{\cdot}008$ inches for pitch blocks up to 4·000 ins.

$= 0{\cdot}012$ inches for pitch blocks above this diameter.

All other dimensions required for manufacturing pitch blocks, and the position of the tapped holes used for securing the tapes denoted by the angle $\propto$ in Fig. 21, Book II, are usually supplied with each machine by the makers.

When this information is available, therefore, it is advisable to prepare suitable layout sheets, giving the revelant information for each size of machine.

The maximum spiral angle of helical gears which can be ground on the machines previously described is approximately 45° on the Lees Bradner machine and 55° on the Maag machine, although if it is desired to grind gears of a greater angle of spiral, recourse should be made to the possibilities of profile grinding the teeth on a worm thread grinding machine.

This is due to the fact that a spiral gear of very large spiral angle is in fact a worm gear, the only difference being that in the case of a spiral gear the spiral angle is specified, while for the worm the term helix angle is used, although each angle is a complement of 90°.

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25. ⅞″ ,, ,, ,, (30 ton yield).
26. 1″ ,, ,, ,, (30 ton yield).
27. ¾″ ,, ,, ,, (40 ton yield).
28. ⅞″ ,, ,, ,, (40 ton yield).
29. 1″ ,, ,, ,, (40 ton yield).
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31. Moments of Inertia of Built-up Sections (Instructions and Examples)

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